

Functions

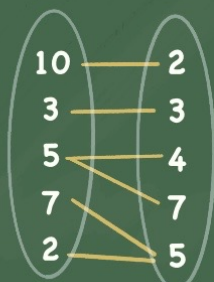
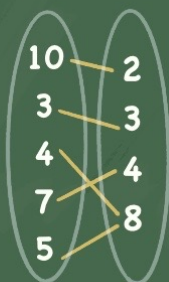
$$f(x) = 2x + 1$$

Input
(domain)

Output
(range)

Function

Not a Function



x	y
-2	-3
-1	-1
0	1
1	3
2	5

domain

range



Question 1

$$f(x) = 3 + 4x$$

$$g(x) = 6x + 7$$

Find, in its simplest form,

(a) $f(3x)$,

$$\begin{aligned} f(3x) &= 3(3+4x) \\ &= 9+12x \end{aligned}$$

[1]

(b) $fg(x)$.

$$\begin{aligned} fg(x) &= f(6x+7) \\ &= 3+4(6x+7) \\ &= 3+24x+28 \\ &= 31+24x \end{aligned}$$

[2]

Question 2

$$f(x) = x^3$$

$$g(x) = 3x - 5$$

$$h(x) = 2x + 1$$

Work out

(a) $ff(2)$,

$$\begin{aligned} ff(2) &= f(2^3) \\ &= f(8) \\ &= (8^3) = 512 \end{aligned}$$

[2]

(b) $gh(x)$ and simplify your answer.

$$\begin{aligned} gh(x) &= g(2x+1) \\ &= 3(2x+1) - 5 \\ &= 6x+3-5 \\ &= 6x-2 \end{aligned}$$

(c) $h^{-1}(x)$, the inverse of $h(x)$.

$$\text{Let } y = h(x) \text{ then } h^{-1}(y) = x$$

[2]

$$\begin{aligned} y &= 2x+1 \\ y-1 &= 2x \\ x &= \frac{y-1}{2} \\ h^{-1}(y) &= \frac{y-1}{2} \\ h^{-1}(x) &= \frac{x-1}{2} \end{aligned}$$

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[2]

Question 3

$$f(x) = 5x - 3$$

$$g(x) = x^2$$

(a) Find $fg(-2)$.

$$\begin{aligned} fg(-2) &= f(-2)^2 \\ &= f(4) \\ &= 5(4) - 3 \\ &= 20 - 3 \\ &= 17 \end{aligned}$$

[2]

(b) Find $gf(x)$, in terms of x , in its simplest form.

$$\begin{aligned} gf(x) &= g(5x - 3) \\ &= (5x - 3)^2 \\ &= 25x^2 - 30x + 9 \end{aligned}$$

[2]

(c) Find $f^{-1}(x)$.

Let $y = f(x)$ then $f^{-1}(y) = x$

$$y = 5x - 3$$

$$\frac{y+3}{5} = x$$

$$f^{-1}(y) = \frac{y+3}{5}$$

$$f^{-1}(x) = \frac{x+3}{5}$$

Question 4

$$f(x) = 3x - 2$$

$$g(x) = \frac{2}{x+1}, \quad x \neq -1$$

(a) Find $gf(2)$.

$$\begin{aligned} gf(2) &= g(3(2) - 2) \\ &= g(4) \\ &= \frac{2}{4+1} = \frac{2}{5} \end{aligned}$$

[2]

(b) Solve $g(x) = 10$.

$$g(x) = 10$$

$$\frac{2}{x+1} = 10$$

$$2 = 10x + 10$$

$$10x = 12$$

$$x = \frac{12}{10} = \frac{6}{5}$$

[2]

(c) Simplify.

$$f(2x) - f(x+2)$$

$$2(3x - 2) - (3x - 2 + 2)$$

$$6x - 4 - 3x + 1 - 2$$

$$3x - 4$$

[3]

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Question 5

$$f(x) = (x-3)^2$$

$$g(x) = \frac{x-1}{4}$$

$$h(x) = x^3$$

Find

(a) $hf(1)$,

$$\begin{aligned} hf(1) &= h(1-3)^2 \\ &= h(4) \\ &= (4)^3 = 64 \end{aligned}$$

[2]

(b) $g^{-1}(x)$,

$$\begin{aligned} \text{Let } y &= g(x) \text{ then } g^{-1}(y) = x \\ y &= \frac{x-1}{4} \rightarrow 4y+1 = x \rightarrow g^{-1}(y) = 4y+1 \\ g^{-1}(x) &= 4x+1 \end{aligned}$$

[2]

(c) $gh(x)$,

$$\begin{aligned} gh(x) &= g(x^3) \\ &= \frac{x^3-1}{4} \end{aligned}$$

[1]

(d) the solution to the equation $f(x) = 0$.

$$\begin{aligned} (x-3)^2 &= 0 \\ x^2 - 6x + 9 &= 0 \\ (x-3)(x-3) &= 0 \\ x &= 3 \end{aligned}$$

[1]

Question 1

$$f(x) = 2x + 3 \quad g(x) = x^2$$

(a) Find $fg(6)$.

$$\begin{aligned} fg(6) &= f(6)^2 \\ &= f(36) \\ &= 2(36) + 3 \\ &= 72 + 3 = 75 \end{aligned}$$

[2]

(b) Solve the equation $gf(x) = 100$.

$$\begin{aligned} gf(x) &= 100 \\ g(2x+3) &= 100 \\ (2x+3)^2 &= 100 \end{aligned}$$

$$\begin{aligned} 4x^2 + 12x + 9 &= 100 \\ 4x^2 + 12x - 91 &= 0 \\ (2x-7)(2x+13) &= 0 \\ x &= \frac{7}{2} \quad \text{or} \quad x = -\frac{13}{2} \end{aligned}$$

[3]

(c) Find $f^{-1}(x)$.

$$\begin{aligned} \text{Let } y &= f(x) \text{ then } f^{-1}(y) = x \\ y &= 2x + 3 \\ y - 3 &= 2x \\ x &= \frac{y-3}{2} \\ f^{-1}(y) &= \frac{y-3}{2} \\ f^{-1}(x) &= \frac{x-3}{2} \end{aligned}$$

[2]

(d) Find $ff^{-1}(5)$.

$$\begin{aligned} ff^{-1}(5) &= f\left(\frac{5-3}{2}\right) = f(1) \\ &= 2(1) + 3 = 5 \end{aligned}$$

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[1]

Question 2

$$f(x) = 5x + 4 \quad g(x) = \frac{1}{2x}, \quad x \neq 0 \quad h(x) = \left(\frac{1}{2}\right)^x$$

Find

(a) $fg(5)$, [2]

$$fg(5) = f\left(\frac{1}{2(5)}\right) = f\left(\frac{1}{10}\right) = 5\left(\frac{1}{10}\right) + 4$$

$$= \frac{1}{2} + 4 = 4.5$$

(b) $gg(x)$ in its simplest form, [2]

$$gg(x) = g\left(\frac{1}{2x}\right) = \frac{1}{2\left(\frac{1}{2x}\right)} = x$$

(c) $f^{-1}(x)$, [2]

Let $y = f(x)$ then $f^{-1}(y) = x$

$$y = 5x + 4 \quad f^{-1}(y) = \frac{y-4}{5} \quad f^{-1}(x) = \frac{x-4}{5}$$

(d) the value of x when $h(x) = 8$. [2]

$$\frac{y-4}{5} = x$$

$$h(x) = 8$$

$$\left(\frac{1}{2}\right)^x = 8$$

$$x = -3$$

Question 3

$$f(x) = x + \frac{2}{x} - 3, \quad x \neq 0 \quad g(x) = \frac{x}{2} - 5$$

Find

(a) $fg(18)$, [2]

$$fg(18) = f\left(\frac{18}{2} - 5\right)$$

$$= f(4)$$

$$= 4 + \frac{2}{4} - 3 = \frac{3}{2}$$

(b) $g^{-1}(x)$. [2]

Let $y = g(x)$ then $g^{-1}(y) = x$

$$y = \frac{x}{2} - 5$$

$$y + 5 = \frac{x}{2}$$

$$2(y + 5) = x$$

$$x = 2y + 10$$

$$g^{-1}(y) = 2y + 10$$

$$g^{-1}(x) = 2x + 10$$

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Question 4

$$f(x) = 4(x+1)$$

$$g(x) = \frac{x^3}{2} - 1$$

- (a) Write down the value of x when $f^{-1}(x) = 2$.

let $y = f(x)$ then $f^{-1}(y) = x$

$$y = 4(x+1) \quad \frac{y-4}{4} = x$$

$$f^{-1}(y) = \frac{y-4}{4}$$

$$f^{-1}(x) = \frac{x-4}{4}$$

$$\frac{x-4}{4} = 2$$

$$x-4 = 8$$

$$x = 12$$

[1]

- (b) Find $fg(x)$. Give your answer in its simplest form.

$$fg(x) = f\left(\frac{x^3}{2} - 1\right)$$

$$= 4\left(\frac{x^3}{2} - 1 + 1\right)$$

$$= \frac{4x^3}{2} = 2x^3$$

[2]

- (c) Find $g^{-1}(x)$.

let $y = g(x)$ then $g^{-1}(y) = x$

$$y = \frac{x^3}{2} - 1$$

$$y+1 = \frac{x^3}{2}$$

$$2y+2 = x^3$$

$$x = \sqrt[3]{2y+2}$$

$$g^{-1}(y) = \sqrt[3]{2y+2}$$

$$g^{-1}(x) = \sqrt[3]{2x+2}$$

[3]

Question 1

$$f(x) = x^2 + 1$$

$$g(x) = \frac{x+2}{3}$$

- (a) Work out $ff(-1)$.

$$ff(-1) = f(-1^2 + 1)$$

$$= f(2)$$

$$= 2^2 + 1$$

$$= 5$$

[2]

- (b) Find $gf(3x)$, simplifying your answer as far as possible.

$$gf(3x) = g(3x^2 + 1)$$

$$= \frac{3x^2 + 1 + 2}{3}$$

$$= \frac{3x^2 + 3}{3} = x^2 + 1$$

[3]

- (c) Find $g^{-1}(x)$.

Let $y = g(x)$ then $g^{-1}(y) = x$

$$y = \frac{x+2}{3}$$

$$3y - 2 = x$$

$$g^{-1}(y) = 3y - 2$$

$$g^{-1}(x) = 3x - 2$$

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[2]

Question 2

$$f(x) = 3x + 5 \quad g(x) = 4x - 1$$

(a) Find the value of $gg(3)$.

$$\begin{aligned} gg(3) &= g(12-1) \\ &= g(11) \\ &= 3(11)+5 \\ &= 33+5 \\ &= 38 \end{aligned}$$

[2]

(b) Find $fg(x)$, giving your answer in its simplest form.

$$\begin{aligned} fg(x) &= f(4x-1) \\ &= 3(4x-1)+5 \\ &= 12x-3+5 \\ &= 12x+2 \\ &= 2(6x+1) \end{aligned}$$

[2]

(c) Solve the equation.

$$\begin{aligned} \text{let } y=f(x) \text{ then } f^{-1}(y)=x & \quad f^{-1}(x)=11 \\ y=3x+5 & \quad f^{-1}(y)=\frac{y-5}{3} \\ \frac{y-5}{3}=x & \quad f^{-1}(x)=\frac{x-5}{3} \end{aligned}$$

$$\frac{x-5}{3}=11$$

$$x-5=33$$

$$x=38$$

[1]

Question 3

$$f(x) = \frac{1}{x+4} \quad (x \neq -4)$$

$$g(x) = x^2 - 3x$$

$$h(x) = x^3 + 1$$

(a) Work out $fg(1)$.

$$\begin{aligned} fg(1) &= f(1^2-3(1)) \\ &= f(1-3) \\ &= f(-2) \\ &= \frac{1}{-2+4} = \frac{1}{2} \end{aligned}$$

[2]

(b) Find $h^{-1}(x)$.

$$\begin{aligned} \text{let } y=h(x) \text{ then } h^{-1}(y)=x & \\ y=x^3+1 & \quad h^{-1}(y)=\sqrt[3]{y-1} \\ y-1=x^3 & \quad h^{-1}(x)=\sqrt[3]{x-1} \\ x=\sqrt[3]{y-1} & \end{aligned}$$

[2]

(c) Solve the equation $g(x) = -2$.

$$\begin{aligned} g(x) &= -2 \\ x^2 - 3x &= -2 \\ x^2 - 3x + 2 &= 0 \\ (x-2)(x-1) &= 0 \\ x=2 \text{ (or)} & \quad x=1 \end{aligned}$$

[3]

Question 4

$$f(x) = x^3 \qquad g(x) = 2x - 3$$

(a) Find

$$\begin{aligned} \text{(i) } g(6), &= 2(6) - 3 && [1] \\ &= 12 - 3 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{(ii) } f(2x), &&& [1] \\ f(2x) &= (2x)^3 \\ &= 8x^3 \end{aligned}$$

(b) Solve $fg(x) = 125$. [3]

$$\begin{aligned} fg(x) &= 125 && 2x - 3 = 5 \\ f(2x - 3) &= 125 && 2x = 8 \\ (2x - 3)^3 &= 125 && 2 = 4 \\ 2x - 3 &= \sqrt[3]{125} \end{aligned}$$

(c) Find the inverse function $g^{-1}(x)$. [2]

$$\begin{aligned} \text{Let } y &= g(x) \text{ then } g^{-1}(y) = x \\ y &= 2x - 3 && g^{-1}(y) = \frac{y+3}{2} \\ \frac{y+3}{2} &= x && g^{-1}(x) = \frac{x+3}{2} \end{aligned}$$

Question 5

$$f(x) = x^2 \qquad g(x) = 2^x \qquad h(x) = 2x - 3$$

$$\begin{aligned} \text{(a) Find } g(3), &&& [1] \\ g(3) &= 2^3 \\ &= 8 \end{aligned}$$

(b) Find $hh(x)$ in its simplest form. [2]

$$\begin{aligned} hh(x) &= h(2x - 3) \\ &= 2(2x - 3) - 3 \\ &= 4x - 6 - 3 \\ &= 4x - 9 \end{aligned}$$

(c) Find $fg(x+1)$ in its simplest form. [2]

$$\begin{aligned} fg(x+1) &= f(2^{x+1}) \\ &= (2^{x+1})^2 \\ &= 2^{2x+2} \\ &= 2 \end{aligned}$$

Question 1

$$f(x) = \frac{x}{4} - 3$$

$$g(x) = 6x - 7$$

$$h(x) = 2^x$$

(a) Work out the value of x when $f(x) = -0.5$.

[2]

$$\frac{x}{4} - 3 = -0.5$$

$$\frac{x}{4} = 2.5$$

$$x = 10$$

(b) Find $g^{-1}(x)$.

[2]

Let $y = g(x)$ then $g^{-1}(y) = x$
 $y = 6x - 7$ $g^{-1}(y) = \frac{y+7}{6}$
 $\frac{y+7}{6} = x$ $g^{-1}(x) = \frac{x+7}{6}$

(c) Work out the value of x when $h(x) = f(13)$.

[2]

$$h(x) = f(13)$$

$$2^x = \frac{13}{4} - 3$$

$$2^x = \frac{1}{4}$$

$$2^x = 2^{-2}$$

$$x = -2$$

Question 2

$$f(x) = x^2$$

$$g(x) = \frac{x-3}{2}$$

Find

(a) $f(-5)$,

[1]

$$(-5)^2 = 25$$

(b) $gf(x)$,

[1]

$$gf(x) = g(x^2)$$

$$= \frac{x^2 - 3}{2}$$

(c) $g^{-1}(x)$.

[2]

Let $y = g(x)$ then $g^{-1}(y) = x$

$$y = \frac{x-3}{2}$$

$$2y + 3 = x$$

$$g^{-1}(y) = 2y + 3$$

$$g^{-1}(x) = 2x + 3$$

Question 3

$$f(x) = 5 - 3x$$

(a) Find $f(6)$.

$$\begin{aligned} &= 5 - 3(6) \\ &= 5 - 18 \\ &= -13 \end{aligned}$$

[1]

(b) Find $f(x + 2)$.

$$\begin{aligned} &5 - 3(x + 2) \\ &5 - 3x - 6 \\ &-1 - 3x \end{aligned}$$

[1]

(c) Find $ff(x)$, in its simplest form.

$$\begin{aligned} ff(x) &= f(5 - 3x) \\ &= 5 - 3(5 - 3x) \\ &= 5 - 15 + 9x \\ &= -10 + 9x \end{aligned}$$

[2]

(d) Find $f^{-1}(x)$, the inverse of $f(x)$.

$$\begin{aligned} \text{Let } y &= f(x) \text{ then } f^{-1}(y) = x \\ y &= 5 - 3x & f^{-1}(y) &= \frac{5 - y}{3} \\ 3x &= 5 - y & f^{-1}(x) &= \frac{5 - x}{3} \\ x &= \frac{5 - y}{3} \end{aligned}$$

[2]

Question 4

$$f(x) = 3x + 5 \quad g(x) = x^2$$

(a) Find $g(3x)$.

$$(3x)^2 = 9x^2$$

[1]

(b) Find $f^{-1}(x)$, the inverse function.

$$\begin{aligned} \text{Let } y &= f(x) \text{ then } f^{-1}(y) = x \\ y &= 3x + 5 & f^{-1}(y) &= \frac{y - 5}{3} \\ \frac{y - 5}{3} &= x & f^{-1}(x) &= \frac{x - 5}{3} \end{aligned}$$

[2]

(c) Find $ff(x)$.

Give your answer in its simplest form.

$$\begin{aligned} ff(x) &= f(3x + 5) \\ &= 3(3x + 5) + 5 \\ &= 9x + 15 + 5 \\ &= 9x + 20 \end{aligned}$$

[2]

Question 1

$$f(x) = (x+2)^3 - 5$$

$$g(x) = 2x + 10$$

$$h(x) = \frac{1}{x}, x \neq 0$$

Find

(a) $gf(x)$,

$$\begin{aligned} gf(x) &= g((x+2)^3 - 5) \\ &= 2((x+2)^3 - 5) + 10 \\ &= 2(x^3 + 6x^2 + 12x + 8 - 5) + 10 \end{aligned}$$

$$\begin{aligned} &(x+2)^2(x+2) \\ &= (x^2 + 4x + 4)(x+2) \\ &= x^3 + 2x^2 + 4x^2 + 8x + 4x + 8 \\ &= x^3 + 6x^2 + 12x + 8 \end{aligned} \quad [2]$$

(b) $f^{-1}(x)$,

$$\begin{aligned} &= 2x^3 + 12x^2 + 24x + 6 + 10 \\ &= 2x^3 + 12x^2 + 24x + 16 \end{aligned}$$

[3]

Let $y = f(x)$ then $f^{-1}(y) = x$

$$\begin{aligned} y &= (x+2)^3 - 5 \\ y+5 &= (x+2)^3 \end{aligned}$$

$$\begin{aligned} \sqrt[3]{y+5} &= x+2 \\ \sqrt[3]{y+5} - 2 &= x \end{aligned}$$

$$\begin{aligned} f^{-1}(y) &= \sqrt[3]{y+5} - 2 \\ f^{-1}(x) &= \sqrt[3]{x+5} - 2 \end{aligned}$$

(c) $gh\left(-\frac{1}{5}\right)$.

$$\begin{aligned} gh\left(-\frac{1}{5}\right) &= g\left(-\frac{1}{5}\right) \\ &= g(-5) \\ &= 2(-5) + 10 \\ &= -10 + 10 = 0 \end{aligned}$$

[2]

Question 2

$$f(x) = (x-1)^3$$

$$g(x) = (x-1)^2$$

$$h(x) = 3x + 1$$

(a) Work out $fg(-1)$.

$$\begin{aligned} fg(-1) &= f(-1-1)^2 \\ &= f(4) \\ &= (4-1)^3 \\ &= 27 \end{aligned}$$

[2]

(b) Find $gh(x)$ in its simplest form.

$$\begin{aligned} gh(x) &= g(3x+1) \\ &= (3x+1-1)^2 \\ &= 9x^2 \end{aligned}$$

[2]

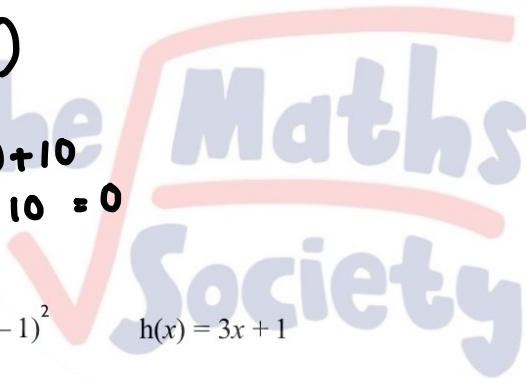
(c) Find $f^{-1}(x)$.

Let $y = f(x)$ then $f^{-1}(y) = x$

$$\begin{aligned} y &= (x-1)^3 \\ \sqrt[3]{y} &= x-1 \\ x &= \sqrt[3]{y} + 1 \end{aligned}$$

$$\begin{aligned} f^{-1}(y) &= \sqrt[3]{y} + 1 \\ f^{-1}(x) &= \sqrt[3]{x} + 1 \end{aligned}$$

[2]



Question 3

(a) $f(x) = 1 - 2x$.

(i) Find $f(-5)$.

$$\begin{aligned} f(-5) &= 1 - 2(-5) \\ &= 1 + 10 \\ &= 11 \end{aligned}$$

[1]

(ii) $g(x) = 3x - 2$.

Find $gf(x)$. Simplify your answer.

$$\begin{aligned} gf(x) &= g(1 - 2x) \\ &= 3(1 - 2x) - 2 \\ &= 3 - 6x - 2 \\ &= 1 - 6x \end{aligned}$$

[2]

(b) $h(x) = x^2 - 5x - 11$.

Solve $h(x) = 0$.

Show all your working and give your answer correct to 2 decimal places.

$$\begin{aligned} x^2 - 5x - 11 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{5 \pm \sqrt{25 - 4(1)(-11)}}{2(1)} \\ &= \frac{5 \pm \sqrt{25 + 44}}{2} \end{aligned}$$

$$\begin{aligned} x &= \frac{5 \pm \sqrt{69}}{2} \\ x &= \frac{5 + \sqrt{69}}{2} \quad (\text{or}) \quad x = \frac{5 - \sqrt{69}}{2} \\ x &= 6.65 \quad (\text{or}) \quad x = -1.65 \end{aligned}$$

[4]

Question 4

$f: x \rightarrow 1 - 2x$ and $g: x \rightarrow \frac{x}{2}$.

(a) Find $fg(7)$.

$$\begin{aligned} fg(7) &= f\left(\frac{7}{2}\right) \\ &= 1 - 2\left(\frac{7}{2}\right) \\ &= -6 \end{aligned}$$

[2]

(b) (i) Solve $f(x) = g(x)$.

$$\begin{aligned} 1 - 2x &= \frac{x}{2} & x &= \frac{2}{5} \\ 2 - 4x &= x \\ 2 &= 5x \end{aligned}$$

[2]

(ii) The graphs of $y = f(x)$ and $y = g(x)$ meet at M .

Find the coordinates of M .

$$\begin{aligned} y &= 1 - 2x \\ y &= 1 - 2\left(\frac{2}{5}\right) \\ &= \frac{1}{5} \end{aligned}$$

$\left(\frac{2}{5}, \frac{1}{5}\right)$

[1]

Question 1

$$f: x \rightarrow 2x - 7 \qquad g: x \rightarrow \frac{1}{x}$$

Find

$$\begin{aligned} \text{(a) } fg\left(\frac{1}{2}\right), \quad f\left(\frac{1}{2}\right) &= f(2) \\ &= 2(2) - 7 \\ &= 4 - 7 \\ &= -3 \end{aligned}$$

[2]

(b) $gf(x)$,

$$\begin{aligned} gf(x) &= g(2x - 7) \\ &= \frac{1}{2x - 7} \end{aligned}$$

[1]

(c) $f^{-1}(x)$.

Let $y = f(x)$ then $f^{-1}(y) = x$

$$y = 2x - 7$$

$$\frac{y+7}{2} = x$$

$$f^{-1}(y) = \frac{y+7}{2}$$

$$f^{-1}(x) = \frac{x+7}{2}$$

[2]

Question 2

$$f(x) = x^2 + 2$$

$$g(x) = (x+2)^2$$

$$h(x) = 3x - 5$$

Find

(a) $gf(-2)$,

$$\begin{aligned} g(-2^2 + 2) &= g(6) \\ &= (6+2)^2 = 64 \end{aligned}$$

[2]

(b) $h^{-1}(22)$.

Let $y = h(x)$ then $h^{-1}(y) = x$

$$y = 3x - 5$$

$$\frac{y+5}{3} = x$$

$$h^{-1}(y) = \frac{y+5}{3}$$

$$h^{-1}(x) = \frac{x+5}{3}$$

[2]

Question 3

$$f(x) = 4x + 1 \quad g(x) = x^3 + 1 \quad h(x) = \frac{2x+1}{3}$$

(a) Find the value of $gf(0)$.

$$\begin{aligned} g(0+1) &= g(1) \\ &= 1^3 + 1 \\ &= 2 \end{aligned}$$

[2]

(b) Find $fg(x)$. Simplify your answer.

$$\begin{aligned} fg(x) &= f(x^3 + 1) \\ &= 4(x^3 + 1) + 1 \\ &= 4x^3 + 4 + 1 \\ &= 4x^3 + 5 \end{aligned}$$

[2]

(c) Find $h^{-1}(x)$.

Let $y = h(x)$ then $h^{-1}(y) = x$

$$y = \frac{2x+1}{3} \quad h^{-1}(y) = \frac{3y-1}{2}$$

$$\frac{3y-1}{2} = x \quad h^{-1}(x) = \frac{3x-1}{2}$$

[2]

Question 4

$$f(x) = \cos x^\circ, \quad g(x) = 2x + 4.$$

Find

(a) $f(60)$, $= \cos 60^\circ$

$$= \frac{1}{2}$$

[1]

(b) $fg(88)$,

$$\begin{aligned} fg(88) &= f(2(88)+4) \\ &= f(180) \\ &= \cos 180^\circ \\ &= -1 \end{aligned}$$

[2]

(c) $g^{-1}(f(x))$.

[2]

Let $y = g(x)$ then $g^{-1}(y) = x$

$$y = 2x + 4 \quad g^{-1}(y) = \frac{y-4}{2}$$

$$\frac{y-4}{2} = x \quad g^{-1}(x) = \frac{x-4}{2} \quad g^{-1}(f(x)) = \frac{\cos x - 4}{2}$$

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Question 1

- The function $f(x)$ is given by

$$f(x) = 3x - 1.$$

Find, in its simplest form,

(a) $f^{-1}f(x)$,

Let $y = f(x)$ then $f^{-1}(y) = x$

$$y = 3x - 1 \quad f^{-1}(y) = \frac{y+1}{3}$$

$$\frac{y+1}{3} \quad f^{-1}(x) = \frac{x+1}{3}$$

(b) $ff(x)$.

$$\begin{aligned} ff(x) &= f(3x-1) \\ &= 3(3x-1) - 1 \\ &= 9x - 3 - 1 \\ &= 9x - 4 \end{aligned}$$

[1]

[2]

Question 2

$f: x \mapsto 5 - 3x$.

(a) Find $f(-1)$.

$$\begin{aligned} 5 - 3(-1) &= 5 + 3 \\ &= 8 \end{aligned}$$

[1]

(b) Find $f^{-1}(x)$.

Let $y = f(x)$ then $f^{-1}(y) = x$

$$y = 5 - 3x$$

$$x = \frac{5-y}{3}$$

$$y - 5 = -3x$$

$$f^{-1}(y) = \frac{5-y}{3}$$

$$-y + 5 = 3x$$

$$f^{-1}(x) = \frac{5-x}{3}$$

$$\frac{-y+5}{3} = x$$

(c) Find $ff^{-1}(8)$.

$$ff^{-1}(8) = f\left(\frac{5-8}{3}\right)$$

$$= f(-1)$$

$$= 5 - 3(-1)$$

$$= 5 + 3$$

$$= 8$$

[2]

[1]

Question 3

$$f(x) = \frac{x+3}{x}, x \neq 0.$$

(a) Calculate $f\left(\frac{1}{4}\right)$.

$$= \frac{\frac{1}{4} + 3}{\frac{1}{4}} = 13$$

[1]

(b) Solve $f(x) = \frac{1}{4}$.

$$\frac{x+3}{x} = \frac{1}{4}$$

$$x+3 = \frac{1}{4}x$$

$$x - \frac{1}{4}x = -3$$

$$\frac{3}{4}x = -3$$

$$x = -4$$

[2]

Question 4

$$f(x) = 10^x.$$

(a) Calculate $f(0.5)$.

$$10^{0.5} = 3.16$$

[1]

(b) Write down the value of $f^{-1}(1)$.

$$\text{Let } y = 10^x \text{ then } f^{-1}(y) = x$$

$$y = 10^x$$

$$x = \log y$$

$$f^{-1}(y) = \log y$$

$$f^{-1}(x) = \log x$$

[1]

Question 5

$$f(x) = \frac{x+1}{2} \text{ and } g(x) = 2x + 1.$$

(a) Find the value of $gf(9)$.

$$\begin{aligned} g\left(\frac{9+1}{2}\right) &= g(5) \\ &= 2(5) + 1 \\ &= 11 \end{aligned}$$

[1]

(b) Find $gf(x)$, giving your answer in its simplest form.

$$\begin{aligned} gf(x) &= g\left(\frac{x+1}{2}\right) \\ &= 2\left(\frac{x+1}{2}\right) + 1 \\ &= x + 1 + 1 \\ &= x + 2 \end{aligned}$$

[2]

(c) Solve the equation $g(x)^{-1} = 1$.

$$\text{Let } y = g(x) \text{ then } g^{-1}(y) = x$$

$$y = 2x + 1$$

$$\frac{y-1}{2} = x$$

$$g^{-1}(y) = \frac{y-1}{2}$$

$$g^{-1}(x) = \frac{x-1}{2}$$

$$\frac{x-1}{2} = 1$$

$$x-1 = 2$$

$$x = 3$$

[2]

Question 6

$$f: x \rightarrow 2x - 1 \text{ and } g: x \rightarrow x^2 - 1.$$

Find, in their simplest forms,

(a) $f^{-1}(x)$,

$$\text{Let } y = f(x) \text{ then } f^{-1}(y) = x$$

$$y = 2x - 1 \quad f^{-1}(y) = \frac{y+1}{2}$$

$$\frac{y+1}{2} = x \quad f^{-1}(x) = \frac{x+1}{2}$$

[2]

(b) $gf(x)$.

$$\begin{aligned} gf(x) &= g(2x-1) \\ &= (2x-1)^2 - 1 \\ &= 4x^2 - 4x + 1 - 1 \\ &= 4x^2 - 4x \\ &= 4x(x-1) \end{aligned}$$

[2]